Estimating the heat equation

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# Introduction

# Deriving the schemes

## Order of Scheme 1

The one-dimensional heat equation is defined as , where α is the rate of diffusion. The initial conditions gives enough information for . In both schemes, we have . In S1, the one-sided derivative is .

Solving for an iterative formula:

Given our initial conditions , we can solve for any point on defined on .

For S2, the symmetric derivative is .

Solving for an iterative formula:

But this iterative formula has a dependence at for the right-hand side, which we do not have at ! Therefore, we will instead solve for every dt locations using these new equations with a dependence of only . I would like to thank this video for helping me conceptualize the midpoint approximation: <https://www.youtube.com/watch?v=D-huCvF15-g>.

Before we can solve this, we need to estimate in terms of . With the exact formula given for , we have the luxury to set small enough such that the average is sufficient to estimate, such that .

Let us create a system of equations for a given value for all multiples of . Let us assume that we have the solution to for all multiples of dx, the left-hand side evaluates to a scalar. Let . We can generate equations, spanning to . Given that , we have enough equations to solve for to . We can construct a matrix and solve as such, where , the evaluation of the right-hand side.

For , we have the solution to By induction is solvable.

## Order of Scheme 2

Error on both terms can be calculated using Taylor series.

About let

About , let

Plugging in the equations.

Because the degree of error is the same from both sources, to minimize error they should be similar in size.

# Richardson’s Extrapolation

## Derivation

To reduce the degree of error, we apply Richardson’s extrapolation where is the estimation of , the exact value, in the form of:

Where is the estimation of taking steps, and is the refined estimation.

Our degree of error for scheme 2 is , so . We can expand to another term to get:

As such, Richardson’s Extrapolation decreases the degree of error to

## Application

# Graphs

# Conclusion

# Attached Programs